Sensibilidad del Receptor

1- Bit-Error Rate (BER)

2- Potencia mínima recibida

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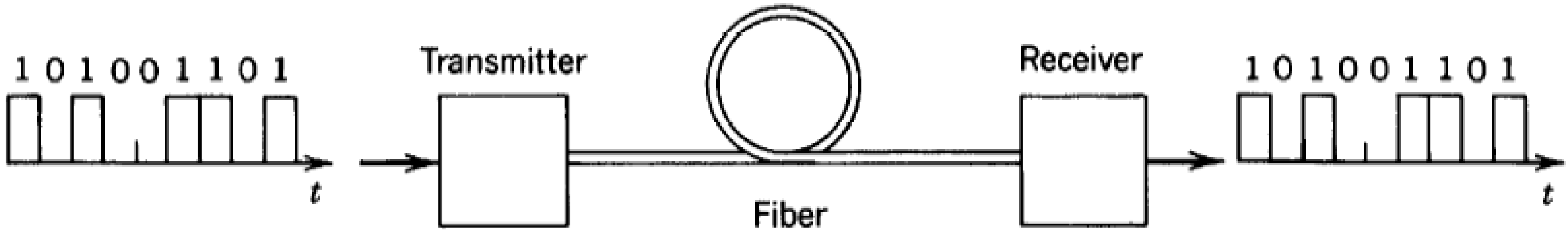
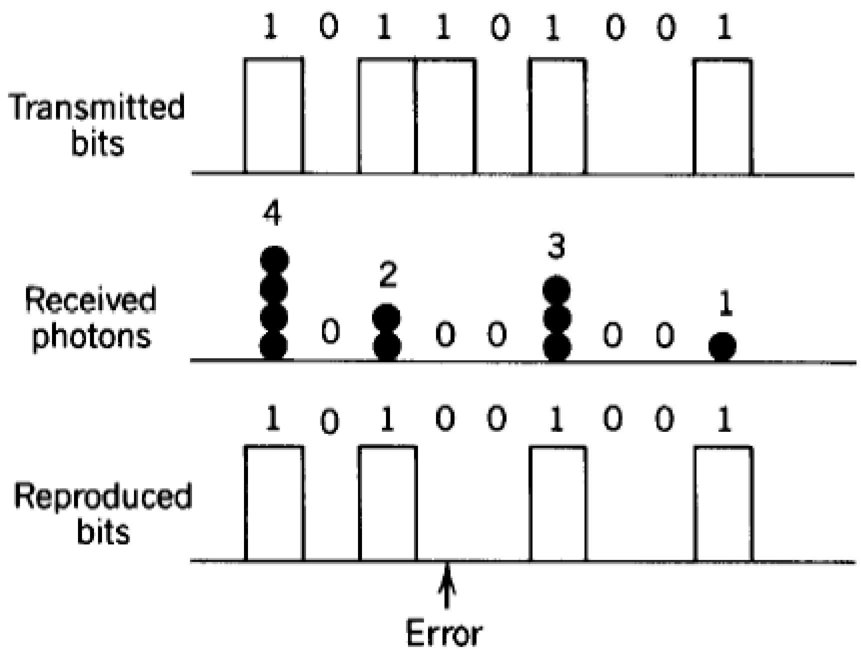
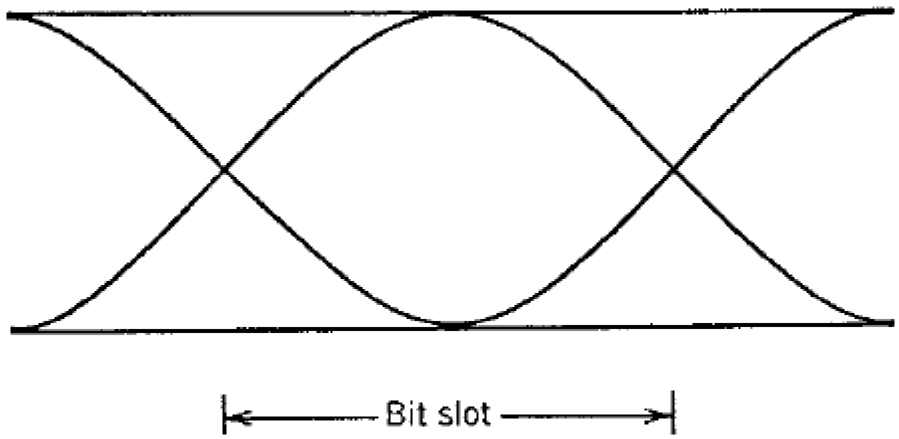
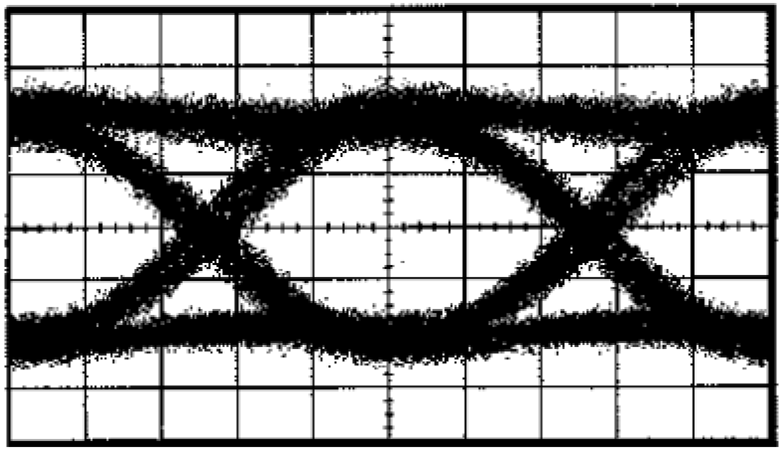
1- Bit-Error Rate (BER)



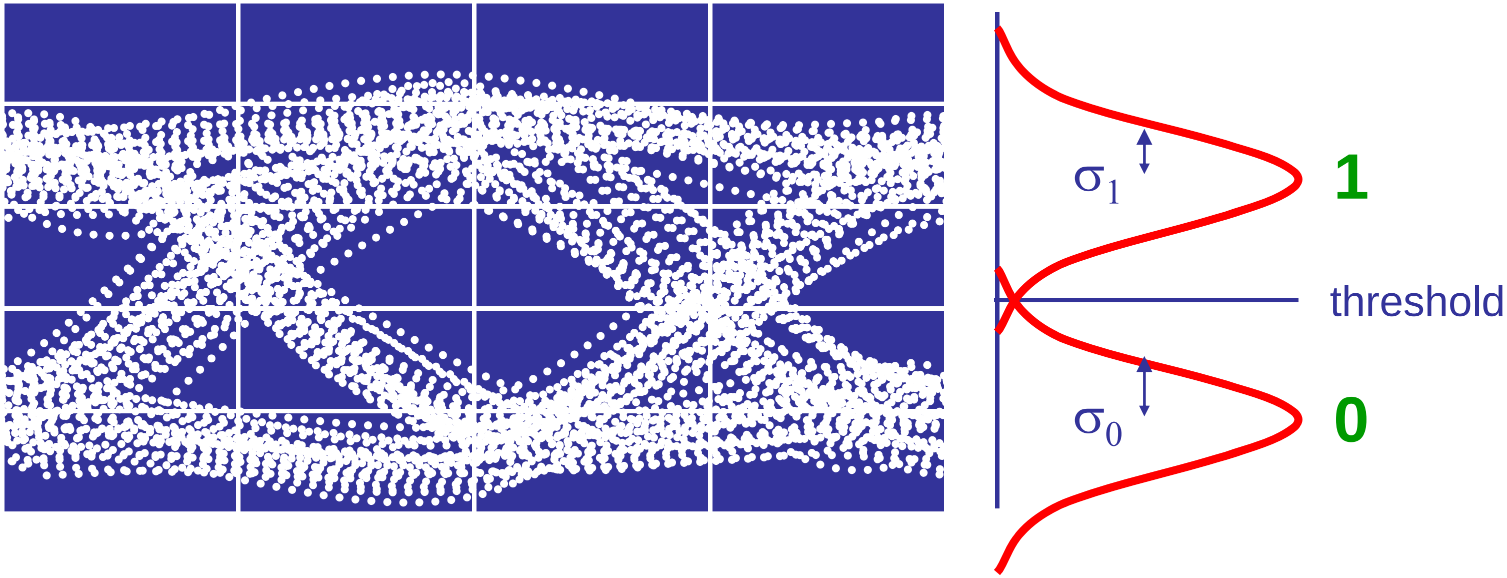


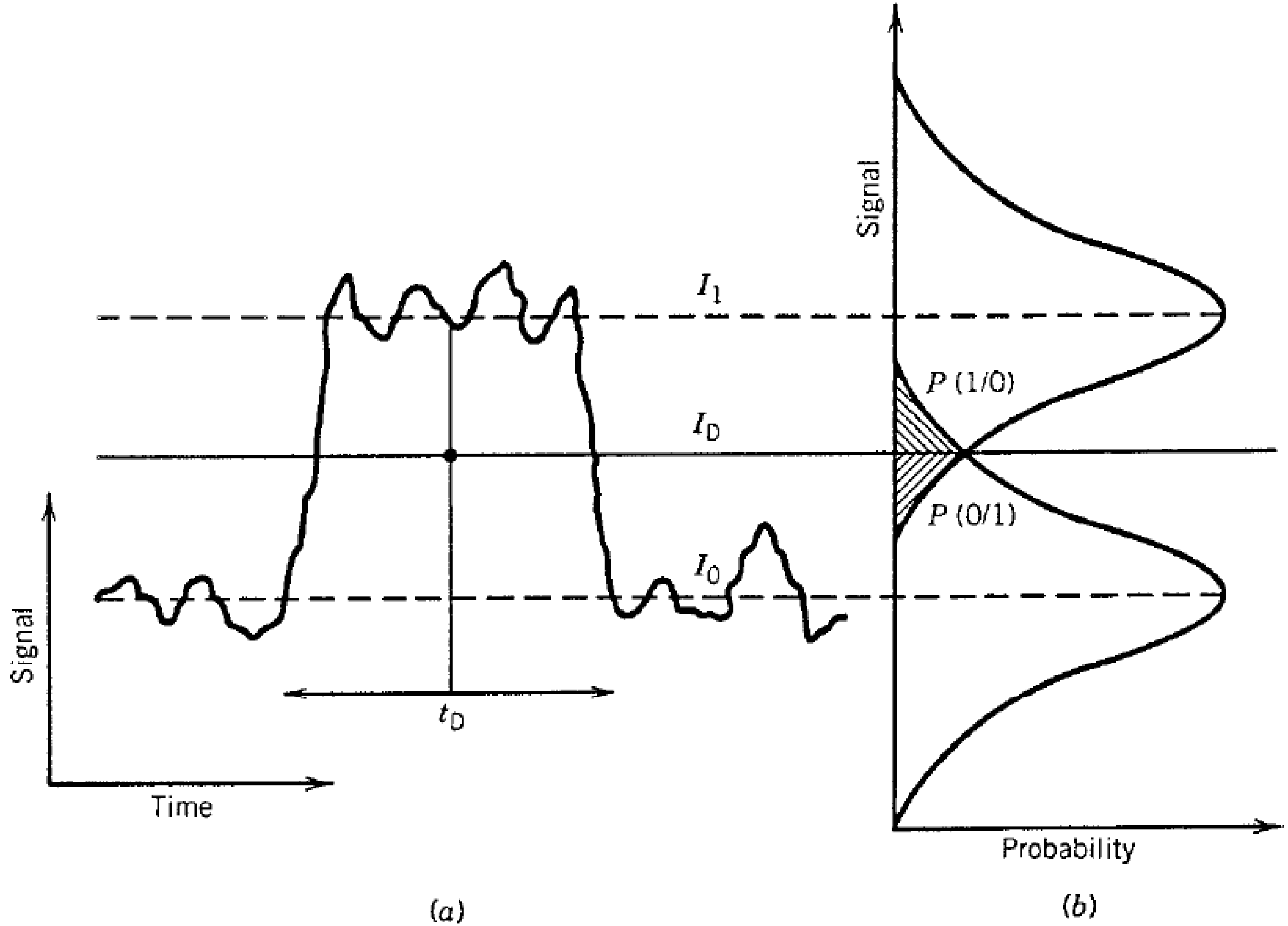
(1)





(1)



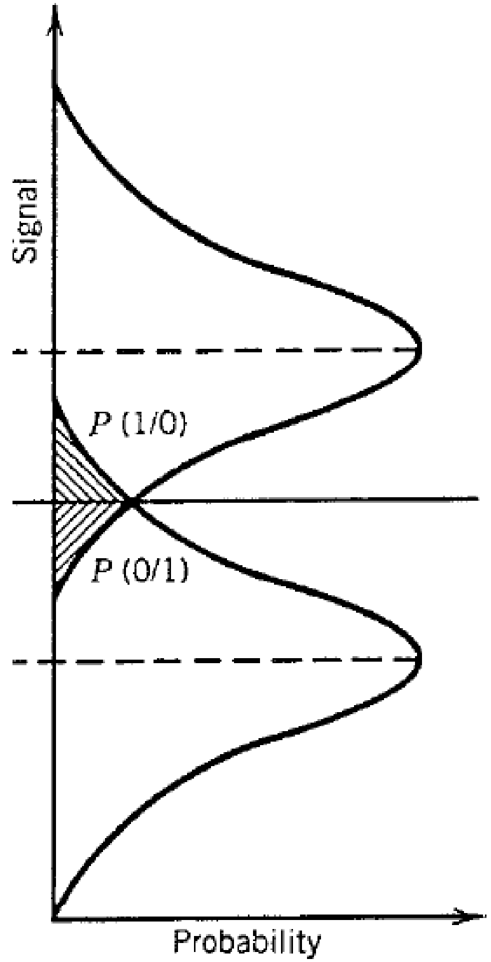


(a) Fluctuating signal generated at the receiver. (b) Gaussian probability densities of 1 and 0 bits. The dashed region shows the probability of incorrect identification. (1)

$$BER = \frac{\text{número de pulsos erróneos}}{\text{número de pulsos enviados}} =$$

$$= \underbrace{p(1)}_{\frac{1}{2}} \; \underbrace{P(0/1)}_{\substack{\text{Probabilidad de} \\ \text{decidir 0,} \\ \text{cuando es 1}}} + \underbrace{p(0)}_{\frac{1}{2}} \; \underbrace{P(1/0)}_{\substack{\text{Probabilidad de} \\ \text{decidir 1,} \\ \text{cuando es 0}}}$$

$$\boxed{BER = \frac{P(0/1) + P(1/0)}{2}}$$



(1)

$p(I)$ depende de la función densidad de probabilidad de las fluctuaciones de las señales de ruido

$$\left.\begin{array}{l}\text{Ruido térmico} \rightarrow \text{estadística gausiana } (\overline{i_T} = 0,\ \sigma^2{}_{i_T}) \\ \text{Ruido shot} \rightarrow \text{estadística poisson, quasi-gausiana } (\overline{i_S} = 0,\ \sigma^2{}_{i_S})\end{array}\right\} \sigma^2 = \sigma^2{}_{i_S} + \sigma^2{}_{i_T}$$

$$erfc(x) = \frac{2}{\sqrt{x}} \int_x^{\infty} e^{-y^2} dy \begin{cases} erfc(0) = 1 \\ erfc(\infty) = 0 \end{cases}$$

$$P(0/1) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{-\infty}^{I_D} e^{-\frac{(I-I_1)^2}{2\sigma_1^2}} dI = \frac{1}{2} erfc\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right) = \frac{1}{2} erfc\left(\frac{Q_1}{\sqrt{2}}\right) \qquad \left[Q_1 = \frac{I_1 - I_D}{\sigma_1}\right]$$

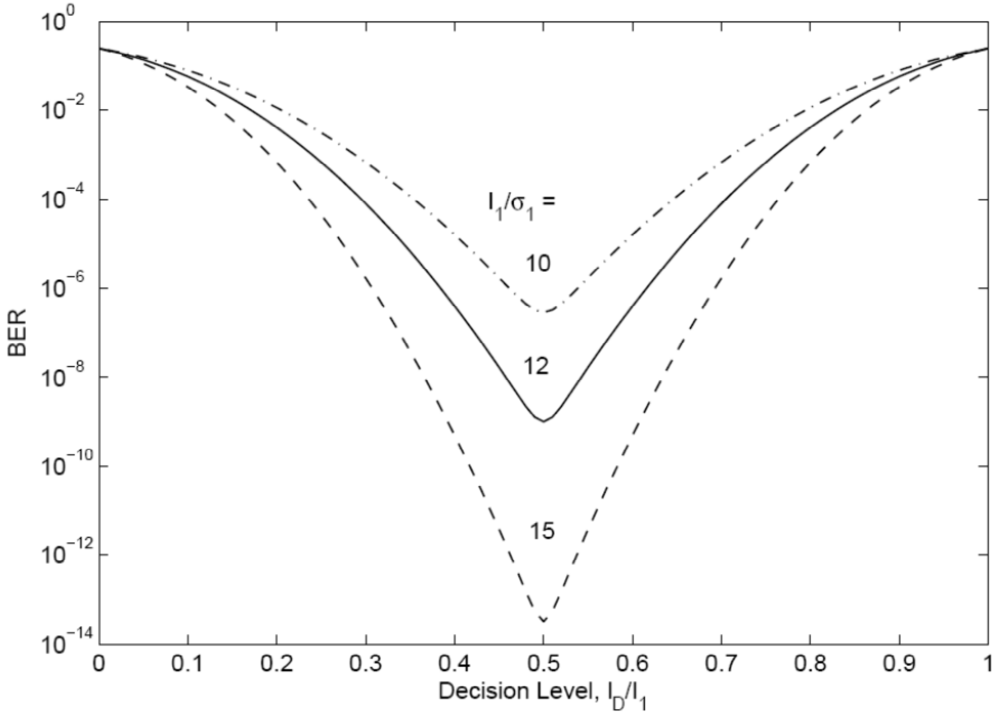
$$P(1/0) = \frac{1}{\sigma_0\sqrt{2\pi}} \int_{I_D}^{\infty} e^{-\frac{(I-I_0)^2}{2\sigma_0^2}} dI = \frac{1}{2} erfc\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right) = \frac{1}{2} erfc\left(\frac{Q_0}{\sqrt{2}}\right) \qquad \left[Q_0 = \frac{I_D - I_0}{\sigma_0}\right]$$

$$\boxed{BER = \frac{1}{4}\left[erfc\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right) + erfc\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)\right]}$$

$$BER_{Mínimo} \rightarrow \frac{d\left(BER\right)}{d\left(I_D\right)} = 0 \rightarrow \underbrace{\frac{I_1 - I_D}{\sigma_1}}_{Q_1} = \underbrace{\frac{I_D - I_0}{\sigma_0}}_{Q_2}$$

$$Q_1 = Q_0 = \boxed{Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}}$$

$$I_D = \frac{\sigma_1 I_0 + \sigma_0 I_1}{\sigma_1 + \sigma_0} \rightarrow \text{Caso particular } \left(\sigma_1 = \sigma_0\right) \rightarrow I_D = \frac{I_1 + I_0}{2}$$



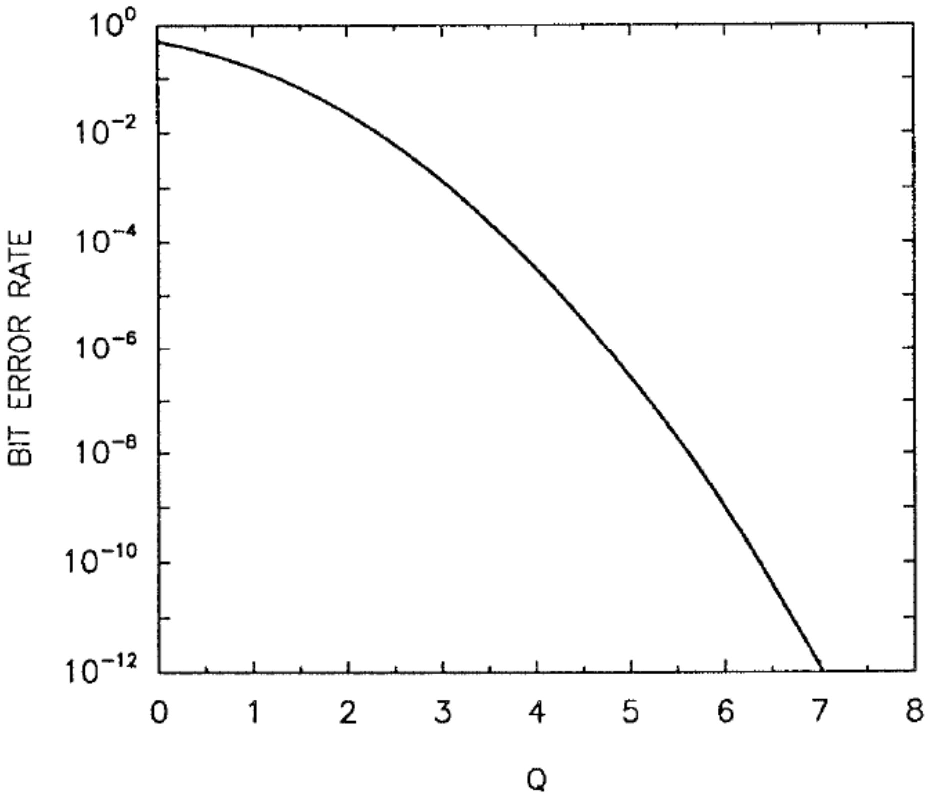
(1)

$$\boxed{BER = \frac{1}{2}\left[erfc\left(\frac{Q}{\sqrt{2}} \right) \right]}$$

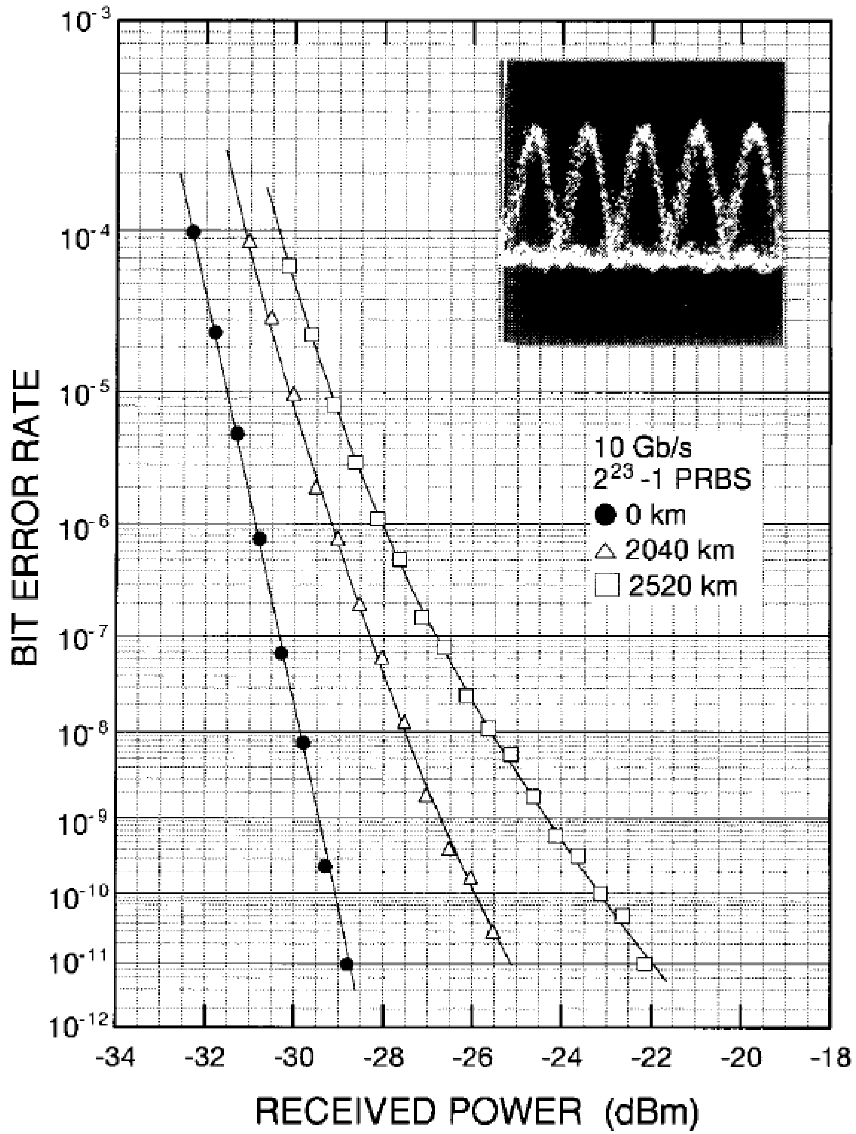
$$Q > 4 \rightarrow BER \approx \frac{1}{\sqrt{2\pi}} \frac{e^{-Q^2/2}}{Q}$$

$$Q = 6 \rightarrow BER = 10^{-9}$$

$$Q = 7 \rightarrow BER = 10^{-12}$$



Bit-error rate versus the Q parameter. (1)



BER curves measured for three fiber-link lengths in a 1.55-μm transmission experiment at 10 Gb/s. Inset shows an example of the eye diagram at the receiver. (1)

2- Potencia mínima recibida

Sensibilidad $\rightarrow$ Potencia mínima promedio $\left[\overline{P}_{rec}\right]$ necesaria para lograr un BER $\rightarrow Q$

$$\text{Señal}\begin{cases}\text{Bit } 1 \rightarrow P_1 \rightarrow I_1 = M\Re P_1 \\ \text{Bit } 0 \rightarrow P_0 = 0 \rightarrow I_0 = 0\end{cases} \quad \boxed{\overline{P}_{rec} = \frac{P_1 + P_0}{2} = \frac{P_1}{2}}$$

$$\text{Ruido}\begin{cases}\text{Bit } 1 \rightarrow \sigma_1 = \sqrt{\sigma_{i_S}{}^2 + \sigma_{i_T}{}^2} = \sqrt{2qM^2 F_{APD}\Re(\underbrace{2\overline{P}_{rec}}_{P_1})\Delta f + \frac{4k_B T}{R_L} F_n \Delta f} \\ \text{Bit } 0 \rightarrow \sigma_0 = \sigma_{i_T}\end{cases}$$

$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0} = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2M\Re\overline{P}_{rec}}{\sqrt{\sigma_{i_S}{}^2 + \sigma_{i_T}{}^2} + \sigma_{i_T}} = \frac{2M\Re\overline{P}_{rec}}{\sqrt{2qM^2 F_{APD}\Re(2\overline{P}_{rec})\Delta f + \sigma_{i_T}{}^2} + \sigma_{i_T}} \rightarrow$$

$\rightarrow$ Despejando se tiene la Sensibilidad $\rightarrow$ $\boxed{\overline{P}_{rec} \approx \frac{Q}{\Re}\left(qF_{APD} Q\Delta f + \frac{\sigma_{i_T}}{M}\right)}$

Casos particulares de sensibilidad

1) Límite ruido térmico) $(\sigma_{i_T}{}^2 \gg \sigma_{i_S}{}^2)$ (PIN) $(M = 1)$

$$\left(\overline{P}_{rec}\right)_{PIN} \approx \frac{Q}{\Re}\left(\cancel{qF_{APD}Q\Delta f} + \sigma_{i_T}\right) = \frac{Q}{\Re}\sigma_{i_T}$$

$$\rightarrow \sigma_{i_T} \propto \sqrt{\Delta f} \rightarrow \underline{\left(\overline{P}_{rec}\right)_{PIN} \propto \sqrt{B}}$$

Ejemplo:

$$\left.\begin{array}{l} BER = 10^{-9} \rightarrow Q = 6 \\ \Re = 1A/W \\ \sigma_{i_T} = 100nA \end{array}\right\} \rightarrow \left(\overline{P}_{rec}\right)_{PIN} = 0,6\mu W \quad (-32,2dBm)$$

$$\left.\begin{array}{l}\text{Señal}\left\{\begin{array}{l}\text{Bit } 1 \rightarrow I_1 \\ \text{Bit } 0 \rightarrow I_0 = 0\end{array}\right\} \\ \text{Ruido}\left\{\begin{array}{l}\text{Bit } 1 \rightarrow \sigma_1 = \sigma_{i_T} \\ \text{Bit } 0 \rightarrow \sigma_0 = \sigma_{i_T}\end{array}\right.\end{array}\right\} Q = \frac{I_1}{2\sigma_{i_T}}$$

$$SNR = \frac{I_1^{\ 2}}{\sigma_{i_T}^{\ 2}}$$

$$\left.\begin{array}{l} Q = \frac{I_1}{2\sigma_{i_T}} \\ SNR = \frac{I_1^{\ 2}}{\sigma_{i_T}^{\ 2}} \end{array}\right\} \underline{SNR = 4Q^2} \rightarrow BER = \frac{1}{2} erfc\left(\frac{\sqrt{SNR}}{2\sqrt{2}}\right)$$

Ejemplo: $BER = 10^{-9} \rightarrow Q = 6 \rightarrow SNR = 4\cdot(6)^2 = 144 \quad (21{,}58 dB)$

2) (Límite ruido shot) $(\sigma_{i_S}{}^2 \gg \sigma_{i_T}{}^2)$ (APD) $(\dfrac{\sigma_{i_T}}{M} \to 0)$

Ideal, límite cuántico

$$\left(\overline{P}_{rec}\right)_{APD} \approx \frac{Q}{\Re}\left(qF_{APD}Q\Delta f + \cancel{\frac{\sigma_{i_T}}{M}}\right) = \frac{qQ^2F_{APD}}{\Re}\Delta f$$

$$\to \underline{\left(\overline{P}_{rec}\right)_{APD} \propto B}$$

$$\text{Ideal} \to F_{APD} = 1 \to \left(\overline{P}_{rec}\right)_{ideal} = \frac{qQ^2}{\Re}\Delta f \to \left(\overline{P}_{rec}\right)_{ideal} \propto B$$

$$\left.\begin{array}{l}\text{Señal}\left\{\begin{array}{l}\text{Bit } 1 \rightarrow I_1 \\ \text{Bit } 0 \rightarrow I_0 = 0\end{array}\right\} \\ \text{Ruido}\left\{\begin{array}{l}\text{Bit } 1 \rightarrow \sigma_1 = \sigma_{i_S} \\ \text{Bit } 0 \rightarrow \sigma_0 = 0\end{array}\right.\end{array}\right\} Q = \frac{I_1}{\sigma_{i_S}}$$

$$SNR = \frac{I_1{}^2}{\sigma_{i_S}{}^2}$$

$$\underline{SNR = Q^2} \rightarrow BER = \frac{1}{2} erfc\left(\frac{\sqrt{SNR}}{\sqrt{2}}\right)$$

Ejemplo: $BER = 10^{-9} \rightarrow Q = 6 \rightarrow SNR = (6)^2 = 36 \quad (15{,}56 dB)$

(1) Agrawal, "Fiber-Optic Communication Systems", 3rd Ed., Wiley, 2002